arXiv:0708.0629v2 [astro-ph] 6 Sep 2007

Analytical description of the Day-Night neutrino asymmetry

A. D. SUPANITSKY[1], J. C. D'OLIVO[1], AND G. A. MEDINA-TANCO[1].
[1]*Departamento de Física de Altas Energías, Instituto de Ciencias Nucleares, Universidad Nacional Autónoma de México, A. P. 70-543, 04510, México, D. F., México.*
dalesupa@gmail.com

Abstract: We present a new treatment of the Earth matter effects on neutrino oscillations that is valid for an arbitrary density profile. When applied to the study of the day-night effect on the solar neutrino flux it renders a simple analytical expression, which is more accurate than those derived by using the perturbation theory and can be extended to higher energies.

Introduction

Different types of experiments have provided compelling evidence for neutrino oscillations [6]. In the case of solar neutrinos the leading effects can be accounted by oscillations between two neutrino flavors, parameterized in terms of the mass square difference $\delta m^2 = m_2^2 - m_1^2$ and the mixing angle θ. A global fit of all the existing data gives $\delta m^2 = (7.9 - 8) \times 10^{-5}$ eV^2 and $\sin^2\theta = 0.310 - 0.315$ [6], which is in good agreement with the results of other groups. These values belong to the region in the parameter space referred to as the Large Mixing Angle Solution (LMA). According to the LMA, the ^{8}B electron neutrinos produced in the Sun undergo a highly adiabatic conversion and are almost totally converted into the mass eigenstate ν_2. Then, the electron neutrino survival probability is $P(\nu_e \to \nu_e) \cong \sin^2\theta$. However, during the night solar neutrinos arriving to terrestrial detectors travel a certain distance through the Earth's matter, which affects the oscillations pattern. This leads to a partial regeneration of the electron neutrino flux, a phenomenon known as the day-night effect.

Matter effects on the neutrino oscillations inside the Earth are conveniently taken into account in terms of the parameter $\varepsilon(t) \equiv 2EV(t)/\delta m^2$, where $V(t) = \sqrt{2} G_F n_e(t)$ represents the potential energy for ν_e, which comes from the charged-current interaction with electrons. Here, G_F is the Fermi constant, E is the neutrino energy, and $n_e(t)$ is the number density of electrons along the neutrino path. In terms of the Avogadro number N_A,

$$\varepsilon(t) \cong 0.019 \left[\frac{E}{10\text{ MeV}}\right]\left[\frac{n_e(t)}{N_A\text{ cm}^{-3}}\right] \times \left[\frac{8 \times 10^{-5}\text{ eV}^2}{\delta m^2}\right], \quad (1)$$

For the favored value of δm^2 and the energy range of solar neutrinos, Earth's density is such that $\varepsilon \ll 1$. Taking advantage of this fact, perturbation theory has been applied to derive an analytical expression for the day-night rate asymmetry to first order in ε [1, 5], which is valid for any density profile. The method simplifies the numerical calculations and it has been subsequently improved by means of a second order expansion in ε [4]. In this work we show that a convenient alternative to the perturbative approach is provided by the Magnus expansion of the evolution operator [3] and from it we derive a more accurate formula for the regeneration probability.

Neutrino Oscillations in Matter

We consider a system consisting of two neutrino flavors, $\Psi_f = (\Psi_e, \Psi_\mu)$, which are related to the mass eigenstate, $\Psi_{mass} = (\Psi_1, \Psi_2)$, according to

$$\Psi_f = U(\theta)\Psi_{mass}, \quad (2)$$

where,

$$U(\theta) = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}. \tag{3}$$

The evolution operator of the system satisfies the equation

$$i\frac{d\mathcal{U}}{dt}(t,t_0) = H(t)\,\mathcal{U}(t,t_0)\,, \tag{4}$$

with the initial condition $\mathcal{U}(t_0,t_0) = \mathbb{1}$. The hamiltonian in the mass base is given by

$$\begin{aligned} H(t) &= \begin{pmatrix} 0 & 0 \\ 0 & \frac{\delta m^2}{2E} \end{pmatrix} + \\ &\qquad V(t)\begin{pmatrix} \cos^2\theta & \sin\theta\cos\theta \\ \sin\theta\cos\theta & \sin^2\theta \end{pmatrix}, \end{aligned} \tag{5}$$

and its eigenvalues are

$$\lambda_\pm(t) = \frac{1}{2}[V(t) + \frac{\delta m^2}{2E} \pm \Delta_m(t))]\,, \tag{6}$$

with

$$\Delta_m(t) = \frac{\delta m^2}{2E}\sqrt{(\varepsilon(t) - \cos 2\theta)^2 + \sin^2 2\theta}\,. \tag{7}$$

Let us now write

$$\begin{aligned} \mathcal{U}(t,t_0) &= \mathcal{P}(t,t_0)\,\mathcal{U}_\mathcal{P}(t,t_0), \\ \mathcal{P}(t,t_0) &= \begin{pmatrix} e^{-i\alpha_-(t,t_0)} & 0 \\ 0 & e^{-i\alpha_+(t,t_0)} \end{pmatrix}, \end{aligned} \tag{8}$$

where $\alpha_\pm(t,t_0) = \int_{t_0}^t dt'\lambda_\pm(t')$. The operator $\mathcal{U}_\mathcal{P}(t,t_0)$ obeys Eq. (4) but for the Hamiltonian $H_\mathcal{P}(t,t_0) = \mathcal{P}^\dagger(t,t_0)[H(t) - H_D(t)]\mathcal{P}(t,t_0)$, where $H_D(t) = diag(\lambda_-(t),\lambda_+(t))$. By expanding $\lambda_\mp$ to first order in $\varepsilon(t)$ we obtain an approximated expression for $H_\mathcal{P}$ with vanishing elements in the diagonal:

$$H_\mathcal{P}(t,t_0) \cong V(t)\frac{\sin 2\theta}{2}\begin{pmatrix} 0 & e^{-i\phi_{t_0\to t}} \\ e^{i\phi_{t_0\to t}} & 0 \end{pmatrix}, \tag{9}$$

with $\phi_{t_0\to t} = \int_{t_0}^t dt'\Delta_m(t')$.

The relevant quantity is the regeneration probability defined as the difference between the day and night probabilities, $F_{reg}(E) \equiv P_{2\to e}(E) - \sin^2\theta$, where $P_{2\to e}(E) = |\langle\nu_e|\hat{\mathcal{U}}(t,t_0)|\nu_2\rangle|^2$. Here, we determine the evolution operator in the mass base from Eq. (8) by evaluating $\mathcal{U}_\mathcal{P}$ in terms of the lowest-order Magnus approximation, $\mathcal{U}_\mathcal{P}(t,t_0) \cong \exp[-i\int_{t_0}^t dt' H_\mathcal{P}(t',t_0)]$. Proceeding in such a way we get

$$\begin{aligned} F_{reg}(E) &= \frac{1}{2}\sin(2I)\sin 2\theta\sin(\phi_{\bar{t}\to t}) \\ &\quad + \sin^2(I)\cos 2\theta, \end{aligned} \tag{10}$$

with

$$I = \sin 2\theta\int_{\bar{t}}^t dt' V(t')\cos(\phi_{\bar{t}\to t'}). \tag{11}$$

In writing Eq. (10), we assumed that the potential is symmetric with respect to the middle point of the trajectory $\bar{t} = (t+t_0)/2$, which is the situation for a medium like the Earth, with a spherically symmetric density profile. By keeping the lowest order terms of the expansion in I, our result for $F_{reg}(E)$ reduces to the one calculated to first order in ε [4].

In order to make a numerical comparison of the different formulas, we examine the case of a neutrino that crosses the Earth passing trough its center. For the electron density we adopt the simplified model called mantle-core-mantle [7]. According to it, $n_e(r)$ is approximated by a step function and the radius of the core and the thickness of the mantle are assumed to be half of the Earth radius. Accordingly, we put

$$n_e(r) = N_A\begin{cases} 5.953\ \mathrm{cm}^{-3}, & r \le R_\oplus/2 \\ 2.48\ \mathrm{cm}^{-3}, & R_\oplus/2 < r \le R_\oplus \end{cases}, \tag{12}$$

where $R_\oplus$ is the radius of the Earth.

Following Ref. [4], we introduce the function

$$\delta(E) = \frac{1}{\bar{F}_{reg}(E)}[F_{reg}^{(appr)}(E) - F_{reg}^{(exact)}(E)], \tag{13}$$

where $F_{reg}^{(appr)}$ is given by a certain (approximated) analytical expression, $F_{reg}^{(exact)}$ is obtained from the exact (numerical) solution, and

$$\bar{F}_{reg}(E) = \frac{1}{2}\varepsilon(t_s)\sin^2\theta \tag{14}$$

is the average regeneration factor evaluated at the surface layer. Essentially, δ represents the relative error of the approximated expression.

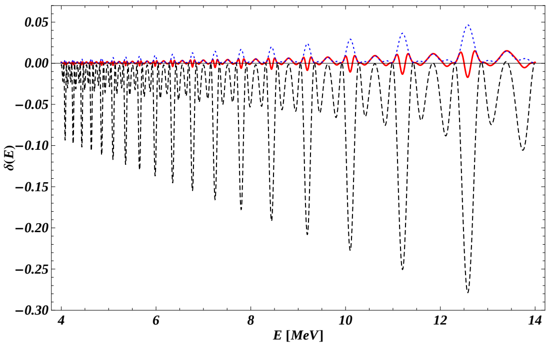

Figure 1: Relative error δ vs the neutrino energy in the case of a neutrino that goes through the Earth passing by its center, for $\delta m^2 = 8 \times 10^{-5}$ eV^2 and $\tan^2\theta = 0.4$. The dashed line and dotted blue line are the first and second order approximations in ε, respectively, and the solid red line corresponds to the first-order Magnus result.

Figure 1 shows δ as a function of the neutrino energy for a neutrino that propagates inside the Earth and goes through its center. $F_{reg}^{(appr)}$ has been computed to first and second order in V and by means of the result given in Eq. (10). From the figure we see that the relative error for the Magnus approximation is always smaller than those corresponding to the perturbative calculations. Although it increases with energy it remains smaller than $\sim 2\%$ for the largest energies of the solar neutrinos.

Day-Night asymmetry

As a function of the energy the day-night asymmetry can be expressed as[1]

$$A_{DN}(E) = \frac{2\,\langle\cos 2\hat{\theta}\rangle F_{reg}}{1 - \langle\cos 2\hat{\theta}\rangle(F_{reg} - \cos 2\theta)}, \quad (15)$$

where,

$$\langle\cos 2\hat{\theta}\rangle(E) = \int_0^{R_\odot} dr f(r) \times \frac{\cos 2\theta - \varepsilon(E,r)}{\sqrt{(\varepsilon(E,r) - \cos 2\theta)^2 + \sin^2 2\theta}}. \quad (16)$$

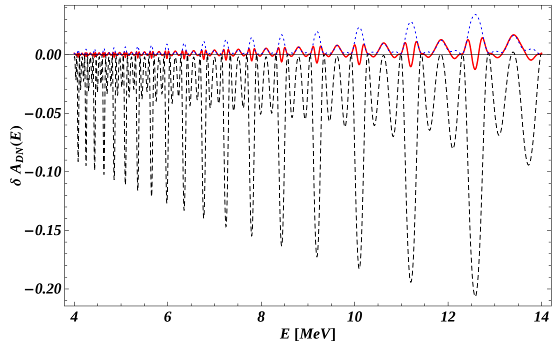

Figure 2: Relative error in the Day-Night asymmetry as a function of the neutrino energy for a neutrino that propagates inside the Earth crossing through the center ($\eta = 0°$). The curves correspond to the first (dashed line) and second (dotted blue line) order in ε and to the Mangus result (solid red line), for $\delta m^2 = 8 \times 10^{-5}$ eV^2 and $\tan^2\theta = 0.4$.

Here, $f(r)$ is the spatial distribution function of the solar neutrino sources [2] and $\varepsilon(E,r)$ is determined by Eq. (1) with $n_e(r)$ now representing the electron density within the Sun [2]. Figures 2 and 3 show the relative error in $A_{DN}(E)$ as a function of the energy for the three approximations examined here and a neutrino trajectory with nadir angle $\eta = 0°$ (neutrino passing through the Earth center) and $\eta = 30°$ (neutrino passing tangent to the core region), respectively. We used the function $f(r)$ corresponding to the ^{8}B neutrinos and in both cases the smallest relative error is obtained with our expression for the regeneration probability. We also see that for all the approximations the relative error is smaller for $\eta = 30°$, which is due to the fact that the electron density, and therefore ε, is smaller in the mantle region of the Earth.

Finally, we also calculate the integrated day-night asymmetry,

$$A_{DN} = 2\int_{E_{th}}^{\infty} dE\phi_\nu(E)\langle\cos 2\hat{\theta}\rangle(E)F_{reg}(E) \times\left[1 - \int_{E_{th}}^{\infty} dE\phi_\nu(E)\langle\cos 2\hat{\theta}\rangle(E) \times\Big(F_{reg}(E) - \cos 2\theta\Big)\right]^{-1}, \quad (17)$$

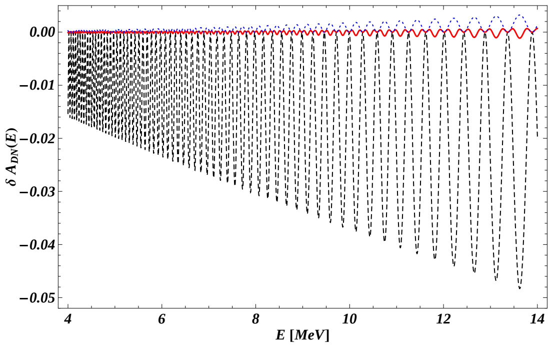

Figure 3: Relative error in the Day-Night asymmetry as a function of the neutrino energy for a neutrino that propagates inside the Earth passing tangent to the core region ($\eta = 30°$). The curves correspond to the first (dashed line) and second (dotted blue line) order in ε and to the Mangus result (solid red line), for $\delta m^2 = 8 \times 10^{-5}$ eV^2 and $\tan^2\theta = 0.4$.

where $\phi_\nu(E)$ is the normalized flux of solar ^{8}B neutrinos and $E_{th} = 5$ MeV is the detection energy threshold for Super-Kamiokande and SNO. Figure 4 shows the relative error in A_{DN} as a function of the cosine of the nadir angle for the three approximated formulas. It can be seen that there are two regions: one corresponding to the propagation in the mantle, $0 < \cos\eta < \sqrt{3}/2$, and the other to the propagation in the mantle and the core, $\cos\eta > \sqrt{3}/2$. The relative error is practically constant in both regions. In the mantle it takes the values -1.7%, 0.07%, and -0.001% for the first order in ε, the second order in ε, and formula (10), respectively. In the core-mantle the corresponding values are -4.7%, 0.51%, and 0.13%.

Conclusions

In this work we have applied the Magnus expansion of the time evolution operator to find approximated analytical solutions of the system of two neutrino flavors coupled very weakly with matter. From this result we derived new expressions for the regeneration probability and the Day-Night asymmetry which give better approximations to the exact numerical results than those obtained by using a perturbative approach.

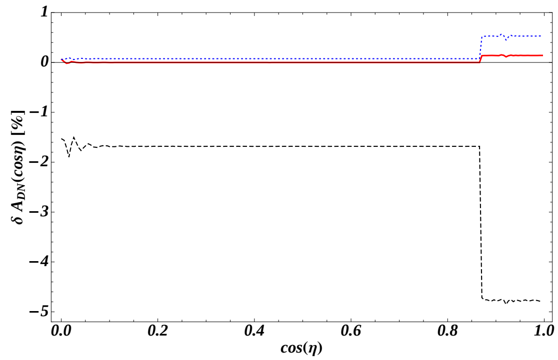

Figure 4: Relative error in the integrated Day-Night asymmetry as a function of the nadir angle. The curves correspond to the first (dashed line) and second (dotted blue line) order in ε and to the Mangus result (solid red line), for $\delta m^2 = 8\times10^{-5}$ eV^2 and $\tan^2\theta = 0.4$.

Acknowledgements

This work has been partially suported by CONACYT Grant 46999-F and by PAPIIT-UNAM Grants IN115607 and IN115707.

References

[1] E. K. Akhmedov, M. A. Tórtola, and J. W. Valle. *JHEP*, 057:0405, 2004.

[2] J. N. Bahcall. http://www.sns.ias.edu/∼jnb/.

[3] J. C. D'Olivo. *Phys. Rev. D*, 45:924, 1992.

[4] A. N. Ioannisian, N. A. Kazarian, A. Y. Smirnov, and D. Wyler. *Phys. Rev. D*, 71:033006, 2005.

[5] A. N. Ioannisian and A. Y. Smirnov. *Phys. Rev. Lett.*, 93:241801, 2004.

[6] A. Yu. Smirnov,. *J. Physics (Conf. Ser.)*, 53:44, 2006.

[7] F. D. Stacey. *Physics of the Earth*. (Wiley, New York), 1977.